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Incompressible Unsteady Flow through a Tube of Variable Cross-Section

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A relationship connecting the rate of change of the volume flow rate of an inviscid incompressible fluid through a tube of varying cross-section with the pressure difference between two arbitrary cross-sections, should prove useful in hydraulic applications and has been used to simulate piston extrusion in a hypervelocity launcher.

The flow is assumed to take place through a uni-directional tube whos axis will be taken as the x axis, and whose cross-section area is given by a suitably differentiable quantity $A(x)$. In addition to incompressibility and zero viscosity, the absence of gravity and other body forces is assumed. While the varying cross-section area prevents the situation from being purely one-dimensional, we use the "quasi-one-dimensional" approach of Rudinger[1] assuming that: a) the column of fluid is long in relation to its cross-section, b) the cross-section area A is a slowly varying function of x.

The flow velocity u and the pressure, p, are thus assumed to be functions of x and the time t. To derive the relation under discussion† we use the Eqs. of conservation of mass and momentum:

$$(Au)_x = 0; \, Au = f(t) \tag{1}$$

$$u_t + uu_x + Vp_x = 0 \tag{2}$$

where V is the (constant) specific volume.
Thus

$$u_t = f'(t)/A(x) = -(\tfrac{1}{2}u^2)_x - Vp_x \tag{3}$$

Integrating (3) between 2 arbitrary cross-sections, x_1 and x_2, with t fixed, gives

$$f'(t)\int_{x_1}^{x_2}\frac{dx}{A(x)} = -\tfrac{1}{2}(u_2^2 - u_1^2) + V(p_1 - p_2)$$

$$= -\tfrac{1}{2}f(t)^2[A_2^{-2} - A_1^{-2}] + V(p_1 - p_2) \tag{4}$$

the subscripts 1 and 2 denoting values at x_1 and x_2, respectively. Solving (4) for $f'(t)$

$$f'(t) = \frac{V(p_1 - p_2) - \tfrac{1}{2}f(t)^2[A_2^{-2} - A_1^{-2}]}{\int_{x_1}^{x_2}\frac{dx}{A(x)}} \tag{5}$$

The previous Eq. has been found useful in the simulation of light gas hypervelocity launchers, specifically the simulation of the extrusion of a plastic piston, after reaching a region of decreasing cross-section area. While conditions a) and b) previously mentioned may not be satisfied, this approach was felt to give a reasonable approximation as part of the total launcher simulation.

Clearly Eq. (5) can be used to determine the piston motion from the pressures at its 2 ends. This in turn allows the use of the same Eq. for determining the pressure at intermediate points. The possibility of obtaining negative internal pressures should be born in mind. This would probably result in the longitudinal separation of the material.

Equation (5) might also be applied to transient problems in hydraulics. In the case of steady flow, in which $f'(t) \equiv 0$. Eq. (5) given the Bernouilli relation,

$$Vp_1 + \tfrac{1}{2}u_1^2 = Vp_2 + \tfrac{1}{2}u_2^2 \tag{6}$$

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Index categories: LV/M Simulation; Hydrodynamics; Nozzle and Channel Flow.

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†The relation (equation (5)) is believed to have been originated by the late Mario Cloutier at DREV-who developed it for the light-gas gun application discussed below. The author is grateful to the Associate Editor for the particular derivation given here.

Numerical Investigation of Leading-Edge Vortex for Low-Aspect Ratio Thin Wings

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I. Introduction

FOR plane and cambered delta wings and for wings with curved leading edges exhibiting the well-known leading-edge vortex flow phenomenon,[1-5] we present theoretical results obtained by a singularities method.[6] These results are compared with flow visualizations performed in a water-tunnel.

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